

Long-Distance Entanglement Distribution with Single-Photon Sources

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We present an efficient architecture for quantum repeaters based on single-photon sources in combination with quantum memories for photons. Errors inherent to previous repeater protocols using photon-pair sources are eliminated, leading to a significant gain in efficiency. We establish the requirements on the single-photon sources and on the photon detectors.

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Entangled state distribution over long distances is a challenging task due to the limited transmission efficiencies of optical fibers. To overcome this problem, quantum repeaters are likely to be required [1]. The basic principle of quantum repeaters consists in decomposing the full distance into shorter elementary links. Quantum memories allow the creation of entanglement independently for each link. This entanglement can then be extended to the full distance using entanglement swapping.

The protocol proposed here is similar to the well-known Duan-Lukin-Cirac-Zoller (DLCZ) scheme [2], and to its recent modification based on photon pairs and multi-mode memories (P²M³) [3], in that entanglement for an elementary link is created by the detection of a single photon. However, both protocols rely on sources that create *correlated pairs* of excitations, namely one atomic excitation and one photon in the case of DLCZ, and two photons in the case of P²M³. These correlations allow one to establish entanglement between distant memories based on the detection of a photon which could have come from either of two remote sources.

Our protocol uses *single-photon* sources making it possible to eliminate errors due to two-pair emission events, which are unavoidable for Refs. [2, 3]. This leads to a significant improvement in the achievable entanglement distribution rate. Moreover our scheme is compatible with the use of multi-mode memories [3], spatial and frequency multiplexing [4], and improved entanglement connection [5], all of which promise additional speed-ups.

We begin by recalling the basic principles of the P²M³ protocol. The DLCZ protocol is equivalent for the purposes of the present discussion. The architecture of an elementary link is represented in Fig. 1 (a). The procedure to entangle two remote locations *A* and *B* requires one photon-pair source and one memory at each location. The pair sources are coherently excited such that each of them can emit a pair with a small probability $p/2$, corresponding to the state

$$\left(1 + \sqrt{p/2}(a^\dagger a'^\dagger + b^\dagger b'^\dagger) + O(p)\right)|0\rangle; \quad (1)$$

a, a' and b, b' are the pairs of modes emitted by the

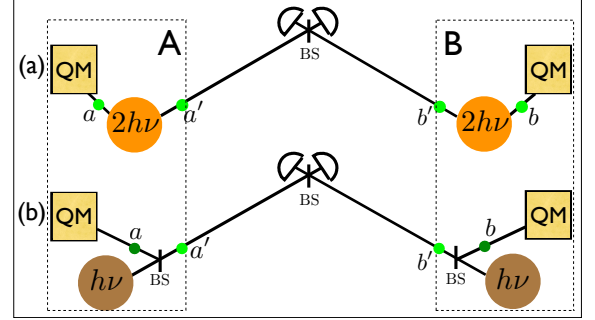


FIG. 1: (Color online) Schematic architecture of an elementary link connecting two locations *A* and *B* for (a) the P²M³ protocol that uses photon pair sources [3], and (b) the new single-photon source protocol. Sources, memories and detectors are represented by circles, squares and half-circles respectively. Vertical bars labeled BS denote beam-splitters. For both (a) and (b), the detection of a single photon behind the central beam-splitter projects the two memories into an entangled state.

sources located at *A* and *B* respectively, and $|0\rangle$ is the vacuum state. The modes a and b are stored in memories close to the respective sources while the modes a' and b' are sent through optical fibers to a station located half-way between *A* and *B*, where they are combined on a beam-splitter. Omitting for simplicity the phase acquired by the photons during their transmission, the modes after the beam-splitter are $\tilde{a} = (a' + b')/\sqrt{2}$, $\tilde{b} = (a' - b')/\sqrt{2}$ (see Ref. [3] for a more complete discussion). The detection of a single photon in mode \tilde{a} , for example, creates the state $\frac{1}{\sqrt{2}}(a^\dagger + b^\dagger)|0\rangle$ which corresponds to a single delocalized excitation. The modes a and b are stored in memories, and the stored single excitation can be written as an entangled state $\frac{1}{\sqrt{2}}(|1_A 0_B\rangle + |0_A 1_B\rangle)$, where $|0_{A(B)}\rangle$ and $|1_{A(B)}\rangle$ denote zero and one photon respectively stored in the memory at *A* (*B*). This entanglement can further be extended to long distances using entanglement swapping [2, 3].

The performance of the described protocol, which has

many attractive features, is limited by a fundamental error mechanism. Even if the pair sources are ideal, i.e. even if they emit at most one pair each, there is a probability $p^2/4$ that two pairs will be emitted in total. If this is the case, and if one photon is lost during its transmission through the fiber or by detector failure, the detection of a single photon in the mode \tilde{a} or \tilde{b} generates the state $|1_A 1_B\rangle$, corresponding to two full memories. This state, which is not the desired entangled state, introduces errors and limits the fidelity of the created entanglement. To preserve high fidelity, one has to use sources with low emission probability $p \ll 1$, such that the probability to get simultaneous emissions at A and B is sufficiently small. This limits the achievable distribution rate of entangled states. The same problem occurs for the DLCZ protocol, which, for the purpose of the present discussion, differs from the P²M³ protocol only by the fact that the modes a and b are created directly in the memories [2, 3].

The proposed new scheme using single-photon sources is free of these fundamental errors. The architecture of our scheme is represented in Fig. 1 (b). The two remote locations contain each one single-photon source and one memory. When they are excited, each of the two sources ideally creates one photon. The photons created at A and B are sent through identical beam splitters with reflection and transmission coefficients α and β satisfying $|\alpha|^2 + |\beta|^2 = 1$, such that after the beam-splitters, the state of the two photons is $(\alpha a^\dagger + \beta a'^\dagger)(\alpha b^\dagger + \beta b'^\dagger)|0\rangle$, which can be rewritten as

$$(\alpha^2 a^\dagger b^\dagger + \alpha\beta (a'^\dagger b^\dagger + a^\dagger b'^\dagger) + \beta^2 a'^\dagger b'^\dagger)|0\rangle. \quad (2)$$

The modes a, b are stored in memories. The modes a', b' are coupled into optical fibers and combined on a beam splitter at a central station, with the modes after the beam-splitter denoted by \tilde{a} and \tilde{b} as before. We are interested in the detection of one photon, for example in the mode \tilde{a} . Let us consider the contributions from the three terms in Eq. (2). The term $a^\dagger b^\dagger|0\rangle$ which corresponds to two full memories, cannot generate the expected detection and thus does not contribute to the entanglement creation. The term $(a'^\dagger b^\dagger + a^\dagger b'^\dagger)|0\rangle$ may induce the detection of a single photon in mode \tilde{a} with probability $\alpha^2 \beta^2 \eta_t \eta_d$, where η_t is the efficiency of transmission to the central station, and η_d is the single-photon detection efficiency. For this term, the detection of a photon in \tilde{a} creates the desired state $\frac{1}{\sqrt{2}}(a^\dagger + b^\dagger)|0\rangle$ associated to entangled memories. Note that in contrast with the P²M³ protocol, the entanglement creation uses correlations between modes $a'-b$ and $a-b'$ rather than correlations between $a-a'$ and $b-b'$ (see relevant term in Eq. (1)). Finally the term $a'^\dagger b'^\dagger|0\rangle$ may also produce a single photon in mode \tilde{a} if one of the two photons is lost. The probability to produce a single detection in mode \tilde{a} for this term is approximately $\beta^4 \eta_t \eta_d$, since for long distances $\eta_t \ll 1$ [6]. This detection creates the vacuum state $|0\rangle$

for the remaining modes a and b . The state created by the detection of a single photon in mode \tilde{a} is thus given by

$$\beta^2 |0\rangle\langle 0| + \alpha^2 |\psi\rangle\langle \psi| \quad (3)$$

where $|\psi\rangle = \frac{1}{\sqrt{2}}(a^\dagger + b^\dagger)|0\rangle$. The state $|\psi\rangle$ corresponds to an entangled state of the two memories located at A and B , $\frac{1}{\sqrt{2}}(|1_A 0_B\rangle + |0_A 1_B\rangle)$, while the vacuum state $|0\rangle$ corresponds to $|0_A 0_B\rangle$, i.e. both memories are empty. We emphasize that none of the three terms in Eq. (2) leads to a component of the form $|1_A 1_B\rangle$. This is a crucial difference compared to the pair-source protocols (DLCZ and P²M³).

The further steps are as for the DLCZ protocol [2, 3]. Neighboring links are connected via entanglement swapping, creating an entangled state $\frac{1}{\sqrt{2}}(|1_A 0_Z\rangle + |0_A 1_Z\rangle)$ between two distant locations A and Z . Moreover each location contains two memories, denoted A_1 and A_2 for location A etc. Entangled states of the given type are established between A_1 and Z_1 , and between A_2 and Z_2 . By post-selecting the case where there is one excitation in each location, one generates an effective state of the form

$$\frac{1}{\sqrt{2}}(|1_{A1} 1_{Z2}\rangle + |1_{A2} 1_{Z1}\rangle). \quad (4)$$

The vacuum component in Eq. (3) does not contribute to this final state, since if one of the two pairs of memories contains no excitation, it is impossible to detect one excitation in each location. The vacuum components thus have no impact on the fidelity of the final state. This is not the case for components involving two full memories as in Refs. [2, 3], which may induce one excitation in each location and thus decrease the fidelity. Note that vacuum components, which exist for the single-photon source protocol already at the level of the elementary links, occur for the pair-source protocols as well, starting after the first entanglement swapping procedure [2].

In contrast to the P²M³ protocol, the wavelength of the photons stored in the memory is necessarily the same as that of the traveling photons, requiring memories that operate in the wavelength range where losses in optical fibers are minimal. Such memories could be realized e.g. in Erbium-doped crystals [7], based on an off-resonant Raman process [8], electromagnetically induced transparency [9], or controlled reversible inhomogeneous broadening [10].

The pair-source protocols require a fixed phase relationship between the two pair sources, i.e. between the $a^\dagger a'^\dagger$ and $b^\dagger b'^\dagger$ terms in Eq. (1) [3]. There is no equivalent requirement for the single-photon source protocol, since the phase between the $a'^\dagger b^\dagger$ and $a^\dagger b'^\dagger$ terms in Eq. (2) depends only on the beam-splitter transformation, and not on the phase of the pump laser.

It is important for all considered protocols that the photons from the two sources are indistinguishable, and that the fiber lengths are stable on the time-scale of the entanglement creation for an elementary link [3].

As we have indicated before, the absence of fundamental errors proportional to the entanglement creation probability leads to very significantly improved entanglement distribution rates for the single-photon source protocol. We now discuss this improvement quantitatively. The time required for a successful creation of an entangled state of the form (4) is given by [3]

$$T_{\text{tot}} = \left(\frac{3}{2}\right)^{n+1} \frac{L_0}{c} \frac{1}{P_0 P_1 \dots P_n P_{\text{pr}}}. \quad (5)$$

where $L_0 = L/2^n$ is the length of an elementary link, L is total distance and n is the nesting level of the repeater; P_0 is the success probability for entanglement creation in an elementary link; P_i (with $i \geq 1$) is the success probability for entanglement swapping at the i -th level, and P_{pr} is the probability for a successful projection onto the state Eq. (4).

For the DLCZ protocol, the success probability for entanglement creation in an elementary link is $P_0 = p\eta_t\eta_d$, while for the new single-photon source protocol $P_0 = 2p_1\beta^2\eta_t\eta_d$. Here p_1 is the probability that the source emits one photon ($p_1 = 1$ in the ideal case). The weight of the vacuum component at each nesting level is larger in the single-photon source protocol, and thus the success probabilities P_i (with $i \geq 1$) for entanglement swapping are somewhat lower. However, the probability P_0 can be made much larger than in the photon-pair source protocols. Overall, this leads to higher entanglement distribution rates, as we detail now.

One can show from Eq. (5) that the total time required for entanglement distribution with the single-photon protocol is [12]

$$T_{\text{tot}} = \frac{3^{n+1}}{2} \frac{L_0}{c} \frac{\prod_{k=1}^n (2^k - (2^k - 1)p_1\alpha^2\eta)}{\eta_d\eta_t p_1^{n+3} \beta^2 \alpha^{2n+4} \eta^{n+2}}. \quad (6)$$

Here $\eta = \eta_m\eta_d$, where η_m is the memory efficiency, and we assume photon-number resolving detectors with efficiency η_d ; $\eta_t = \exp(-L_0/(2L_{\text{att}}))$ is the fiber transmission efficiency, and $c = 2 \times 10^8$ m/s is the photon velocity in the fiber. To evaluate the potential performance of our scheme, we calculate the average total time for an entangled state distribution for a distance $L = 1000$ km with $L_{\text{att}} = 22$ km, corresponding to photons at the telecommunications wavelength of $1.5 \mu\text{m}$. We assume $\eta_m = \eta_d = 0.9$. High-efficiency memories [11] and photon-number-resolving detectors [13, 14, 15] are currently being developed. From Eq. (6) one can show that the optimal nesting level for our protocol for these parameter values is $n = 3$, corresponding to 8

elementary links. Assuming $p_1 = 0.95$ and optimizing the formula (6) over β gives $T_{\text{tot}} = 250$ s with $\beta^2 = 0.11$. For the DLCZ protocol, T_{tot} is also minimal for 8 links. The calculation of the errors due to double-pair emission shows that in order to achieve a final fidelity $F = 0.9$ one has to choose $p = 0.003$, leading to $T_{\text{tot}} = 4600$ s. The new single-photon source protocol thus reduces the average time for a successful distribution of an entangled state over 1000 km by a factor of 18. This gain can be understood by comparing the values of P_0 for the two protocols. For the new protocol, $P_0 = 0.01$, whereas for the DLCZ protocol $P_0 = 0.0001$. The difference between the calculated factor 18 and the ratio between the P_0 values is due to the lower success probabilities for entanglement swapping.

Distance	DLCZ (s)	n	SPS (s)	n	β^2	gain
1000 km	4600	3	250	3	0.11	18
1500 km	28400	3	1560	3	0.11	18
2000 km	156700	3	6000	4	0.08	26
2500 km	650000	4	15300	4	0.08	42

TABLE I: Average times for entanglement distribution over various distances for the DLCZ and single-photon source (SPS) protocols. The optimal nesting level n and beam-splitter transmission coefficient β^2 are given. We assume high-efficiency memories and photon-detectors ($\eta_m = \eta_d = 0.9$). For the DLCZ protocol, the fidelity of the final state constrains the probability p of photon-pair emission to be small, e.g. for $F=0.9$, one has to choose $p = 0.003$ for both 1000 and 1500 km and $p = 7 \times 10^{-4}$ for both 2000 and 2500 km. In the case of single-photon source protocol, the fidelity is not fundamentally limited by the success probability of single-photon emission p_1 , chosen to be equal to 0.95. As shown in the last column, the gain for the single-photon source protocol compared to the DLCZ protocol increases from a factor of 18 for 1000 km to a factor of 42 for 2500 km. Note that the indicated average times could further be decreased by several orders of magnitude using e.g. multi-mode memories [3].

Different distances from 1000 to 2500 km are considered and the corresponding gain with respect to the DLCZ protocol is shown in Table I. This gain increases with the distance. For example, it reaches a factor larger than 40 for 2500 km. The proposed protocol thus improves the entanglement distribution over long distances very significantly.

In our examples, we have chosen $p_1 = 0.95$. The single-photon source protocol achieves an advantage over the DLCZ protocol as soon as $p_1 > 0.67$, for all the considered distances between 1000 km and 2500 km. Efficient sources are thus required for profiting from the proposed protocol, cf. below.

Our architecture is compatible with the use of multi-mode memories. The indicated average times for

entanglement creation can thus be reduced by several orders of magnitude depending on the number of modes the memory can store [3]. Spatial and frequency multiplexing [4] and entanglement swapping by two-photon detection [5] could further increase the distribution rates.

We have shown that the single-photon protocol has no fundamental error mechanism, i.e. the fidelity of the created entangled states will be equal to one as long as all components of the architecture work perfectly. However, imperfections do affect the fidelity. We have studied two kinds of imperfection that are likely to be relevant for implementations, namely detector dark counts and a small probability for a single-photon source to emit two photons.

We first discuss dark counts, i.e. detector clicks in the absence of photons. A dark count of one of the detectors located at the central station can be associated with two full memories, if the photons emitted by the two sources located at A and B are in the modes a and b . The corresponding state $|1_A 1_B\rangle$ does not coincide with the expected entangled state and thus decreases the fidelity. One can show by explicit calculation that the fidelity of the final state compared to the ideal state (4) for a repeater with 8 elementary links is [12]

$$F = 1 - 16 \left[\frac{25}{\beta^2 p_1} - (25\eta - 1) \left(\frac{1}{p_1} - 1 \right) \right] \frac{p_{\text{dark}}}{\eta_t \eta_d} \quad (7)$$

to first order in $\frac{p_{\text{dark}}}{\eta_t \eta_d}$, where p_{dark} is the probability for a detector to give a dark count. Note that one only has to take into account the effects of dark counts during the creation of entanglement for the elementary links, where fiber transmission losses are very large and real detection probabilities correspondingly low. For the entanglement swapping steps of the protocol the detectors are located close to the memories and the real detection probabilities are much larger than realistic dark count probabilities. We evaluate the constraints on the dark counts by considering the set of parameter values used before for a distance of 1000 km. One finds that in order to achieve $F = 0.9$, p_{dark} has to be smaller than 4.6×10^{-6} . This seems realistic. Transition-edge sensor detectors can already resolve telecom-wavelength photons of 4 ns duration at a repetition rate of 50 KHz, with an efficiency of 0.88 and negligible noise [14]. In the long run, NbN detectors promise to resolve even shorter pulses at higher rates [15].

We now evaluate the tolerance of our scheme with respect to undesired two-photon emissions by the sources. Two-photon emissions cause errors that are similar in nature to those for dark counts. Such emissions might be due e.g. to pump laser scattering. One shows by explicit calculation that for a repeater with 8 links the fidelity is

given by [12]

$$F = 1 - 2 \left(\frac{376}{p_1} - (1 - \beta^2)(395\eta - 19) \right) \frac{p_2}{p_1} \quad (8)$$

to first order in p_2 , where p_2 is the probability for each source to emit two photons. For the same values as above one finds that p_2 has to be smaller than 3.7×10^{-4} in order to achieve a fidelity $F = 0.9$.

Single-photon sources as required for the presented protocol, i.e. with high probability of single-photon emission and low probability of two-photon emission can be realized with a variety of approaches. For first demonstration experiments, the most promising approach may be the use of asynchronous heralded single-photon sources based on parametric down-conversion, where $p_1 > 0.6$ and p_2 of order 10^{-4} have already been achieved at 1.5 μm [16]. Even higher $p_1 > 0.8$ has been reported in Ref. [17] at 780 nm. In the long run, sources based on quantum dots [18] embedded in micro-cavities [19] are likely to offer higher repetition rates, which is important in order to fully profit from multi-mode memories. Quantum dot sources that emit at telecom wavelengths are being developed [20]. Single atoms inside high-finesse cavities [21] are also potential candidates, possibly combined with wavelength-conversion techniques [22] in order to reach telecom wavelengths.

We have proposed a quantum repeater protocol based on single-photon sources that eliminates the fundamental errors due to double-pair emission which limit the performance of previous protocols [2, 3]. It is interesting to note that an important initial motivation for the development of single-photon sources was their application for point-to-point quantum key distribution (QKD), while quantum repeaters were thought to require photon-pair sources. In fact, high-bit-rate point-to-point QKD is achieved more conveniently using weak laser pulses with decoy state protocols [23]. On the other hand, we have shown that single-photon sources are very promising for the implementation of efficient quantum repeaters.

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For an overall efficiency $\eta_i \eta_d \ll 1$, this state leads to a single detection with probability $2\eta_i \eta_d$.

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